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Yang, Longzhi; Shen, Qiang

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Towards Adaptive Interpolative Reasoning

Longzhi Yang and Qiang Shen

Abstract— **Fuzzy interpolative reasoning has been extensively studied due to its ability to enhance the robustness of fuzzy systems and to reduce system complexity. However, during the interpolation process, it is possible that multiple object values for a common variable are inferred which may lead to inconsistency in interpolated results. Such inconsistencies may result from defective interpolated rules or incorrect interpolative transformations. This paper presents a novel approach for identification and correction of defective rules in transformations, thereby removing the inconsistencies. In particular, an assumption-based truth maintenance system (ATMS) is used to record dependencies between reasoning results and interpolated rules, while the underlying technique that the general diagnostic engine (GDE) employs for fault localization is adapted to isolate possible faulty interpolated rules and their associated interpolative transformations. From this, an algorithm is introduced to allow for the modification of the original linear interpolation to become first-order piecewise linear. The approach is applied to a carefully chosen practical problem to illustrate the potential in strengthening the power of interpolative reasoning.**

I. Introduction

Fuzzy rule interpolation, originally proposed in [11], [12], [13], significantly improves the robustness of fuzzy reasoning. When given observations have no overlap with any antecedent values, no rule can be fired in classical inference, fuzzy or not fuzzy. However, interpolative reasoning through a sparse rule base may still obtain certain conclusions and thus improve the applicability of fuzzy models. Also, with the help of fuzzy interpolation, the complexity of a rule base can be reduced by omitting those fuzzy rules which may be approximated from their neighboring rules [9], [10]. A number of important interpolating approaches have been presented in the literature, including [2], [3], [8], [9], [10], [12]. In particular, the scale and move transformation-based approach can handle both interpolation and extrapolation which involve multiple fuzzy rules, with each rule consisting of multiple antecedents. This approach also guarantees the uniqueness as well as normality and convexity of the resulting interpolated fuzzy sets. However, little effort has been made when it comes to truth maintenance [5], [7] and conflict diagnosis [6] in interpolation with sparse rule bases, although such techniques have been widely used in supporting other forms of both classical and fuzzy reasoning.

Briefly, ATMS is a common symbolic reasoning technique used in artificial intelligence which is capable of efficiently keeping track of dependent relations amongst logical deductions. GDE is a popular system for multiple fault diagnosis, originally designed to find faults in physical domains. An essential component of GDE is that for isolation of multiple simultaneous faulty elements efficiently via the use of an ATMS. Each set of these multiple simultaneous fault components is called a candidate. For the present research, if each pair of neighboring rules in the sparse rule base are viewed as a fuzzy reasoning component which takes fuzzy sets as the input and produces another fuzzy set as the output, GDE can be exploited to generate possible component candidates that may have led to the observed inconsistencies. Note that theoretically, inconsistency may indicate contradictions of original observations or failure of rules. As an initial research in this area, this paper only focuses on inconsistencies that are caused by interpolated rules while assuming that given observations and rules are true. In particular, ATMS records the dependencies between an interpolated value and its proceeding fuzzy interpolative reasoning components. From this, GDE manipulates on these sets of dependent components of contradictions to generate all the possible candidates.

It is worth noting that, linear interpolation has been used in all existing fuzzy rule interpolating methods. This is based on the presumption that any relationship between the antecedent variables and the consequent variable is linear. However, this is not always complied rigidly in reality which does lead to inconsistencies. Accordingly, each component in a candidate generated by GDE indicates that the interpolations done by this component (or the corresponding neighboring rules) do not satisfy the presumption. This paper offers a modification approach to correcting defective fuzzy reasoning components by means of refinement of the transformation involved in the interpolation. The overall approach is outlined in Figure 1. Firstly, an interpolative reasoning tool performs inferences on a task and passes the inferred results over each step of interpolation to the ATMS for dependency-recording. Then, the ATMS relays any contradictions as well as their dependencies to the GDE which diagnoses the problem and generates all possible component candidates. After that, a modification process takes place to correct a certain candidate to restore consistency. The working of this approach is illustrated by an application example throughout the paper.

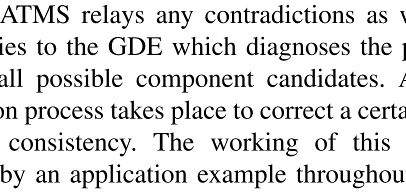

Fig. 1. Adaptive interpolative reasoning process

This paper is structured as follows. Section II describes the relevant background of the scale and move transformation-based fuzzy rule interpolation techniques. Section III shows how to represent fuzzy interpolative reasoning concepts in the framework of ATMS and GDE to generate candidates for modification. Section IV proposes a modification mechanism

Longzhi Yang and Qiang Shen are with the Department of Computer Science, Aberystwyth University, UK (email: {lly07, qqs}@aber.ac.uk).

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for the generated candidates. Section V concludes the paper, with possible further work pointed out.

II. Background of Transformation-based Interpolative Reasoning

A general, scale and move transformation-based fuzzy interpolative reasoning method is presented in [9], [10]. The method is able to handle interpolation of multiple antecedent variables with triangular, complex polygon, Gaussian and other bell-shaped fuzzy membership functions. For the sake of simplicity, only rules involving of a single antecedent variable with triangle shaped membership functions are considered in this paper. However, the underlying ideas can be extended to address more general cases.

Let x_i, $i \in \{1, 2, ..., n\}$, be a variable and $A_{i1}, A_{i2}, ..., A_{im_i}$ be the fuzzy sets in the domain of x_i. Fuzzy set A_{ij} can be represented by a triple (b_{ij}, n_{ij}, e_{ij}), where n_{ij} is the coordinate of the normal point $(\mu_{A_{ij}}(n_{ij}) = 1)$ while b_{ij} and e_{ij} are the coordinates of the start and end points of its support $(\forall x \in (b_{ij}, n_{ij}), \mu_{A_{ij}}(x) > 0)$. If $A_{11} \Rightarrow A_{21}$ and $A_{12} \Rightarrow A_{22}$ are two adjacent fuzzy rules in a sparse rule base, given an observed object value A_{13} of variable x_1, which does not match any existing rule and which is located between fuzzy sets A_{11} and A_{12}, the object value A_{23} of variable x_2 can be derived through fuzzy interpolative reasoning. The interpolation process can be briefly described through the modus ponens interpretation:

$$\frac{\begin{array}{l}\text{O: } x_1 \text{ is } A_{13} \\ R_1\text{: if } x_1 \text{ is } A_{11}, \text{ then } x_2 \text{ is } A_{21} \\ R_2\text{: if } x_1 \text{ is } A_{12}, \text{ then } x_2 \text{ is } A_{22}\end{array}}{\text{C: } x_2 \text{ is } A_{23}}. \quad (1)$$

The two rules used to perform an interpolation are hereafter referred to as the neighboring rules of the interpolated rule which maps an originally uncovered observation to an inferred consequence. To facilitate interpolation, the concept of *representative value* of a triangular fuzzy set A_{ij} is introduced and defined as the center of gravity of the triangle:

$$\text{Rep}(A_{ij}) = \frac{b_{ij} + n_{ij} + e_{ij}}{3}. \quad (2)$$

The *relative placement factor* λ_{ij} of the antecedent (or consequence) A_{ij} of an interpolated rule, with respect to its two neighboring rule antecedents (or consequences) A_{im} and A_{in}, is defined as the the ratio of $d(A_{im}, A_{ij})$ to $d(A_{im}, A_{in})$:

$$\lambda_{ij} = \frac{d(A_{im}, A_{ij})}{d(A_{im}, A_{in})} = \frac{d(\text{Rep}(A_{im}), \text{Rep}(A_{ij}))}{d(\text{Rep}(A_{im}), \text{Rep}(A_{in}))}, \quad (3)$$

where $d(A_{ix}, A_{iy})$ is the distance between fuzzy sets A_{ix} and A_{iy} (given a certain distance metric).

Transformation-based interpolation first constructs an intermediate inference rule $A_{13}' \Rightarrow A_{23}'$ via manipulating the two given adjacent rules $A_{11} \Rightarrow A_{21}$ and $A_{12} \Rightarrow A_{22}$, where A_{13}' and the observation A_{13} have the same *representative value*, and so do A_{23}' and the conclusion A_{23}. Then, the consequence of the intermediate rule A_{23}' is converted into the required fuzzy set A_{23} via scale and move transformations, which are measured by the scale rate and move rate used in order to transform A_{13}' to A_{13}. The procedure of calculating A_{23} is summarized as follows.

1. Calculate the antecedent value of the intermediate rule $A_{13}' = (b_{13}', n_{13}', e_{13}')$ which has the same *representative value* as the observation A_{13}. For this, the *relative placement factor* λ_{13} of the observation A_{13} is calculated first, with respect to its flanks A_{11} and A_{12}:

$$\lambda_{13} = \frac{d(A_{11}, A_{13})}{d(A_{11}, A_{12})} = \frac{d(\text{Rep}(A_{11}), \text{Rep}(A_{13}))}{d(\text{Rep}(A_{11}), \text{Rep}(A_{12}))}. \quad (4)$$

Then,
$$\begin{cases} b_{13}' = (1 - \lambda_{13})b_{11} + \lambda_{13} b_{12} \\ n_{13}' = (1 - \lambda_{13})n_{11} + \lambda_{13} n_{12} \\ e_{13}' = (1 - \lambda_{13})e_{11} + \lambda_{13} e_{12}, \end{cases} \quad (5)$$

which are collectively abbreviated to:

$$A_{13}' = (1 - \lambda_{13})A_{11} + \lambda_{13} A_{12}. \quad (6)$$

2. Calculate the consequence of the intermediate rule A_{23}' by analogy to the calculation of A_{13}' except letting the *relative placement factor* λ_{23} of the conclusion A_{23} be equal to λ_{13}:

$$\lambda_{23} = \lambda_{13}. \quad (7)$$

Then,
$$A_{23}' = (1 - \lambda_{23})A_{21} + \lambda_{23} A_{22}. \quad (8)$$

By the first two steps, the intermediate inference rule $A_{13}' \Rightarrow A_{23}'$ is constructed.

3. Calculate the similarity degree between A_{13}' and A_{13} through two steps of transformation which are measured by scale rate s and move rate m respectively. Let $A_{13}'' = (b_{13}'', n_{13}'', e_{13}'')$ denote the fuzzy set generated by the first step transformation, namely, scale transformation. This transformation transforms the current support (b_{13}', e_{13}') into a new support (b_{13}'', e_{13}'') such that $e_{13}'' - b_{13}'' = e_{13} - b_{13}$, while keeping the *representative value* and the ratio of the left-support (b_{13}'', n_{13}'') to the right-support (n_{13}'', e_{13}'') of the transformed fuzzy set A_{13}'' the same as those of its original. The scale rate s is calculated by:

$$s = \frac{e_{ij}'' - b_{ij}''}{e_{ij}' - b_{ij}'}. \quad (9)$$

It follows from this that

$$\begin{cases} b_{13}'' = \frac{b_{13}'(1+2s) + n_{13}'(1-s) + e_{13}'(1-s)}{3} \\ n_{13}'' = \frac{b_{13}'(1-s) + n_{13}'(1+2s) + e_{13}'(1-s)}{3} \\ e_{13}'' = \frac{b_{13}'(1-s) + n_{13}'(1-s) + e_{13}'(1+2s)}{3}. \end{cases} \quad (10)$$

Move transformation shifts the current fuzzy set support from (b_{13}'', e_{13}'') to (b_{13}, e_{13}) while keeping the same *representative value*, that is, transforming fuzzy set A_{13}'' to fuzzy set A_{13}. The move rate m measures this transformation which is calculated by:

$$\begin{cases} m = \frac{b_{13} - b_{13}''}{\frac{n_{13}'' - b_{13}''}{3}}, & b_{13} \geq b_{13}'' \\ m = \frac{b_{13} - b_{13}''}{\frac{e_{13}'' - n_{13}''}{3}}, & \textit{otherwise}. \end{cases} \quad (11)$$

Given m, if $m \geq 0$, the transformed fuzzy set A_{13} is:

$$\begin{cases} b_{13} = b_{13}'' + m\frac{n_{13}'' - b_{13}''}{3} \\ n_{13} = n_{13}'' - 2m\frac{n_{13}'' - b_{13}''}{3} \\ e_{13} = e_{13}'' + m\frac{n_{13}'' - b_{13}''}{3}. \end{cases} \quad (12)$$

Otherwise, the transformed fuzzy set A_{13} is generated by:

$$\begin{cases} b_{13} = b_{13}'' + m\frac{e_{13}'' - n_{13}''}{3} \\ n_{13} = n_{13}'' - 2m\frac{e_{13}'' - n_{13}''}{3} \\ e_{13} = e_{13}'' + m\frac{e_{13}'' - n_{13}''}{3}. \end{cases} \quad (13)$$

These transformations can be concisely represented by an integrated transformation function T such that the transformation from A_{ij}' to A_{ij} is denoted by $T(A_{ij}', A_{ij})$.

4. Transform A_{23}' to A_{23} with the same transformation function T as used for transforming A_{13}' to A_{13}:

$$T(A_{23}', A_{23}) = T(A_{13}', A_{13}). \quad (14)$$

This ensures that the degree of the similarity between A_{23}' and A_{23} is the same as that between A_{13}' and A_{13}. That is, the more similar A_{13} to A_{13}', the more similar A_{23} to A_{23}'.

III. Minimal Candidate Generation

In fuzzy reasoning, including fuzzy interpolation, it is possible that more than one object value of a single variable are derived or observed. This implies that certain inconsistencies have been reached. For example, variable x is used to illustrate a person's height. It is possible that $x\ is\ tall$ is held in one situation and that $x\ is\ short$ is held in another, while it is contradictory that $x\ is\ tall$ and $x\ is\ short$ are held simultaneously in one single situation, knowing that $tall$ and $short$ represent two semantically different object values.

Given any inconsistency, unless it is caused by two contradictory observations, it can be assumed that the employed fuzzy interpolating method is the only cause of contradiction (if the neighboring rules used are presumed to be true). This is because the consequence of an interpolated rule is generated under the presupposition that the relationship between the antecedent variables and the consequent variable is linear while this is not always to be abided in reality. Following the interpolative approach, each pair of neighboring rules can be seen as a fuzzy reasoning component which takes a fuzzy set as input and produces another fuzzy set as output, as illustrated in Figure 2. Accordingly, a contradiction means that at least one of its dependent fuzzy reasoning components is malfunction unless the original given observations are themselves inconsistent.

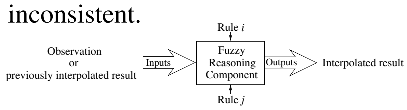

Fig. 2. Fuzzy reasoning component

ATMS can be used to efficiently record the dependencies between a derived proposition and its preceding fuzzy interpolative reasoning components, including those which lead to contradictions. GDE, which is built on the basis of ATMS, can then be employed to generate minimal faulty reasoning component candidates, with each of which explaining the entire set of current contradictions. Here, a minimal candidate is a possible minimal set of defective components which need to be corrected at one time to remove all the contradictions.

A. Contradictions in interpolation

In classical logical reasoning, at a given time, if two unequal values are derived or observed for one single variable in a system, there is a contradiction. The situation varies in fuzzy reasoning because "unequal" in fuzzy representation is a matter of degree. The degree of matching is frequently used to express the extent of equality between two fuzzy sets. Many methods have been proposed to calculate fuzzy matching degrees (e.g. [1], [4]). For computational simplicity, the matching degree between two fuzzy sets A_{ij} and A_{ik}, denoted as $M(A_{ij}, A_{ik})$, in the domain D_{x_i} of variable x_i is herein defined as:

$$M(A_{ij}, A_{ik}) = \sup_{x \in D_{x_i}} [\min(\mu_{A_{ij}}(x), \mu_{A_{ik}}(x))]. \tag{15}$$

Based on this, the degree β of a contradiction with respect to two propositions P $(x_i\ is\ A_{ij})$ and $P'(x_i\ is\ A_{ik})$ can be defined by:

$$\beta = 1 - M(A_{ij}, A_{ik}). \tag{16}$$

A predefined threshold β_0 $(0 \le \beta_0 \le 1)$ is adopted in order to determine those values assigned to a common variable with an unacceptable contradictory degree. A contradiction is called a β_0-*contradiction* if the corresponding degree of contradiction $\beta > \beta_0$.

In fuzzy interpolation, when two or more values of a common variable are obtained, the degree of contradiction between each pair can be calculated by the above definition. The following interpretations will be used in this paper: (i) $\beta = 0$, that is $M(A_{ij}, A_{ik}) = 1$, which means that the two propositions P and P' are not contradictory at all or totally consistent; (ii) $0 < \beta \le \beta_0$, that is $1 - \beta_0 \le M(A_{ij}, A_{ik}) < 1$, which means that the two propositions P and P' are slightly contradictory and the degree of contradiction is tolerable in the system; (iii) $\beta_0 < \beta < 1$, that is $0 < M(A_{ij}, A_{ik}) < 1 - \beta_0$, which means that the two propositions P and P' are seriously contradictory and the degree of contradiction is intolerable in the system; (iv) $\beta = 1$, that is $M(A_{ij}, A_{ik}) = 0$, which means that the two propositions P and P' are totally contradictory, not consistent at all.

B. Representation of interpolation concepts in ATMS

In this work, ATMS is used to record the dependency of the interpolated results as well as contradictions upon those fuzzy reasoning components from which they are inferred. Thus, propositions, contradictions and fuzzy interpolative reasoning components are all represented as ATMS nodes. In addition to the so-called datum field, which trivially denotes a proposition (including the term "false" to represent inconsistency) or a fuzzy reasoning component, an ATMS node has two other fields: justification and label.

1) Justification: A justification describes how a node is derivable from other nodes. Each fuzzy reasoning component is assumed to be initially true and may be detected to be false later. For such a node (i.e. an assumption in classical ATMS terms [6]), its justification just assumes itself to be true. For any given observation O (i.e. a premise in [6]), its corresponding ATMS node has a justification with no antecedent because it is supposed to hold universally, which can be represented as:

$$\Rightarrow O. \tag{17}$$

For any ATMS node with an inferred proposition (i.e. a derived node in [6]), which is obtained through the fuzzy interpolation process as given in (1), can be represented by an ATMS justification as:

$$O, R_iR_j \Rightarrow C, \tag{18}$$

where R_iR_j stands for the fuzzy reasoning component containing the two neighboring rules R_i and R_j $(i \ne j)$ that have been used to infer the outcome C from the observation O.

According to the definition of contradiction above, any two propositions P $(x_i\ is\ A_{ij})$ and $P'(x_i\ is\ A_{ik})$ concerning the same variable x_i are contradictory to a certain degree β. When β is not higher than β_0, the contradictory degree is acceptable and the two considered propositions are treated as being consistent in ATMS. Otherwise, a β_0-$contradiction$ is deduced, which can be represented as:

$$P, P' \Rightarrow_{\beta_0} \bot. \tag{19}$$

2) Label and label-updating: A label is a set of environments each supporting the associated node. An environment contains a minimal set of fuzzy reasoning components that jointly entail the node concerned, thereby describing how the node ultimately depends on those fuzzy reasoning components. An environment is said to be β_0*-inconsistent* if β_0-contradiction is derivable propositionally by the environment and a given justification. An environment is said to be $(1-\beta_0)$*-consistent* if it is not β_0-inconsistent.

The label of each node is guaranteed to be $(1-\beta_0)$-consistent, sound, minimal and complete, except that the label of the special "false" node is β_0-inconsistent rather than $(1-\beta_0)$-consistent. $(1-\beta_0)$*-consistency* means that all environments in the label are at least $(1-\beta_0)$-consistent; $(1-\beta_0)$*-soundness* indicates that the node is derivable from each environment in the label at least to the consistent degree of $(1-\beta_0)$; $(1-\beta_0)$*-minimality* states that the removal of any element from any environment will cause the node underivable from that environment and hence violating the label's $(1-\beta_0)$-soundness; $(1-\beta_0)$*-completeness* implies that every $(1-\beta_0)$-consistent environment from which the node is derivable is a superset of a certain environment in the label, in other words, all minimal $(1-\beta_0)$-consistent environments of the subject node are held within the label.

The label updating algorithm of the ATMS ensures that the above four properties are held all the time. The extended algorithm in this paper is exactly the same as the original one given in [5], except that the environments of a proposition here are at least $(1-\beta_0)$-consistent rather than 1-consistent and that the environments of a contradiction are at least β_0-inconsistent rather than 1-inconsistent (i.e. a contradiction is at least β_0-contradictory rather than 1-contradictory). In particular, the label of the special "false" node gathers all β_0-inconsistent environments. Its corresponding label-updating process is given as follows. Whenever a β_0-contradiction is detected, each environment in its label is added into the label of "false" node and all such environments and their supersets are removed from the label of every other node. Also, any such an environment which is a superset of another is removed from the label of the node "false". Accordingly, the concept of an ATMS context with respect to a $(1-\beta_0)$-consistent environment, is now defined by the collection of the assumptions contained within this environment and of all those nodes that can be derived from these assumptions. Of course, these derived nodes can not be β_0-inconsistent because they are deduced form a $(1-\beta_0)$-consistent environment.

Example 3.1: Suppose that the sparse rule base for a practical problem is given as follows:

R_1: If x_1 is A_{11}, then x_2 is A_{21}; R_2: If x_1 is A_{12}, then x_2 is A_{22};
R_3: If x_2 is A_{23}, then x_3 is A_{31}; R_4: If x_2 is A_{24}, then x_3 is A_{32};
R_5: If x_2 is A_{25}, then x_4 is A_{41}; R_6: If x_2 is A_{26}, then x_4 is A_{42};
R_7: If x_3 is A_{33}, then x_5 is A_{51}; R_8: If x_3 is A_{34}, then x_5 is A_{52};
R_9: If x_4 is A_{43}, then x_5 is A_{53}; R_{10}: If x_4 is A_{44}, then x_5 is A_{54}.

Given $\beta_0 = 0.5$ and three observations, $x_1 = A_{13} = (7.0, 8.0, 9.0)$, $x_1 = A_{14} = (7.6, 8.6, 9.6)$ and $x_4 = A_{45} = (12.0, 13.0, 14.0)$, the interpolation procedures are illustrated in Figure 3 and all the fuzzy sets involved in this example are presented in Figure 4.

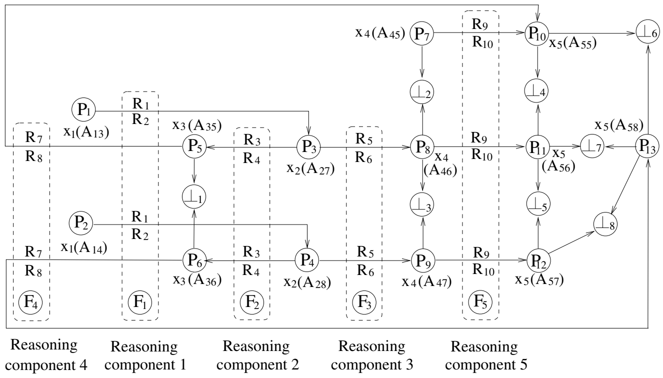

Fig. 3. Discrepancy records in ATMS

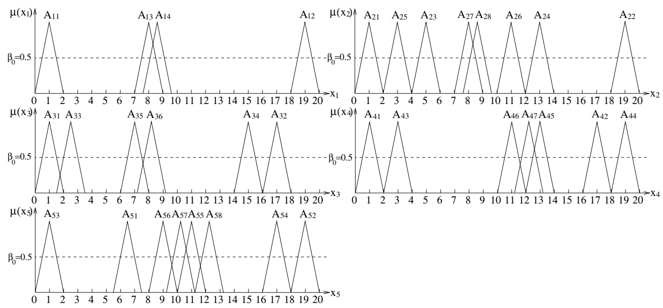

Fig. 4. Fuzzy sets used in the example

In Figure 3, an arrow line flanked by two rules R_i and R_j represents a fuzzy reasoning component, which is denoted as R_iR_j, where R_i and R_j are the neighboring rules used for interpolation. ATMS nodes and contradictions are represented by circles. Particularly, each of F_i, $i \in \{1, 2, ..., 5\}$, is a node denoting a fuzzy reasoning component; each of P_j, $j \in \{1, 2, ..., 13\}$, is a node denoting a proposition; and each of $\perp_k$, $k \in \{1, 2, ..., 8\}$, denotes a β_0-contradiction. These ATMS nodes and contradictions are listed as follows, with all justifications omitted:

$F_1 : \langle R_1R_2, \{\{R_1R_2\}\}\rangle$; $F_2 : \langle R_3R_4, \{\{R_3R_4\}\}\rangle$;
$F_3 : \langle R_5R_6, \{\{R_5R_6\}\}\rangle$; $F_4 : \langle R_7R_8, \{\{R_7R_8\}\}\rangle$;
$F_5 : \langle R_9R_{10}, \{\{R_9R_{10}\}\}\rangle$; $P_1 : \langle x_1 = A_{13}, \{\{\}\}\rangle$;
$P_2 : \langle x_1 = A_{14}, \{\{\}\}\rangle$; $P_3 : \langle x_2 = A_{27}, \{\{R_1R_2\}\}\rangle$;
$P_4 : \langle x_2 = A_{28}, \{\{R_1R_2\}\}\rangle$; $P_5 : \langle x_3 = A_{35}, \{\{R_1R_2, R_3R_4\}\}\rangle$;
$P_6 : \langle x_3 = A_{36}, \{\{R_1R_2, R_3R_4\}\}\rangle$; $P_7 : \langle x_4 = A_{45}, \{\{\}\}\rangle$;
$P_8 : \langle x_4 = A_{46}, \{\{R_1R_2, R_5R_6\}\}\rangle$;
$P_9 : \langle x_4 = A_{47}, \{\{R_1R_2, R_5R_6\}\}\rangle$;
$P_{10} : \langle x_5 = A_{55}, \{\{R_1R_2, R_3R_4, R_7R_8\}, \{R_9R_{10}\}\}\rangle$;
$P_{11} : \langle x_5 = A_{56}, \{\{R_1R_2, R_5R_6, R_9R_{10}\}\}\rangle$;
$P_{12} : \langle x_5 = A_{57}, \{\{R_1R_2, R_5R_6, R_9R_{10}\}\}\rangle$;
$P_{13} : \langle x_5 = A_{58}, \{\{R_1R_2, R_3R_4, R_7R_8\}\}\rangle$;
$\perp_1 : \langle \perp, \{\{R_1R_2, R_3R_4\}\}\rangle$; $\perp_2 : \langle \perp, \{\{R_1R_2, R_5R_6\}\}\rangle$;
$\perp_3 : \langle \perp, \{\{R_1R_2, R_5R_6\}\}\rangle$; $\perp_4 : \langle \perp, \{\{R_1R_2, R_5R_6, R_9R_{10}\}\}\rangle$;
$\perp_5 : \langle \perp, \{\{R_1R_2, R_5R_6, R_9R_{10}\}\}\rangle$;
$\perp_6 : \langle \perp, \{\{R_1R_2, R_3R_4, R_7R_8\}\}\rangle$;
$\perp_7 : \langle \perp, \{\{R_1R_2, R_3R_4, R_5R_6, R_7R_8, R_9R_{10}\}\}\rangle$;
$\perp_8 : \langle \perp, \{\{R_1R_2, R_3R_4, R_5R_6, R_7R_8, R_9R_{10}\}\}\rangle$.

In particular, a specific ATMS node "false", denoted by $P_\perp$, which collectively represents all the contradictions listed above from $\perp_1$ to $\perp_8$, is given as follows:

$P_\perp : \langle \perp, \{\{R_1R_2, R_3R_4\}, \{R_1R_2, R_5R_6\}\}\rangle$.

There are just two minimal environments in the label of the "false" node. This is because all the others are the supersets of at least one of these, which are therefore removed. The label of $P_\perp$ means that at least one element of set $\{R_1R_2, R_3R_4\}$ and one element of set $\{R_1R_2, R_5R_6\}$ are faulty simultaneously. Also, the labels of nodes P_i, $i \in \{5, 6, 8, 9, 11, 12, 13\}$, are empty after the removal of the environments which are the supersets of at least one label environment of the "false" node. In the mean time, node P_{10} is revised to the following: $P_{10} : \langle x_5 = A_{55}, \{\{R_9R_{10}\}\}\rangle$.

C. Minimal candidate generation by GDE

GDE [6] generates minimal candidates by manipulating the label of the specific "false" node. A candidate is a particular set of assumptions which may be responsible for the whole set of current contradictions. Because a β_0-inconsistent environment indicates that at least one of its assumption is faulty, a candidate must have a nonempty intersection with each β_0-inconsistent environment. Thus, each candidate is constructed by taking one assumption from each environment in the label of "false" node. Supersets removal then ensures such generated candidates to be minimal. In light of this, a successful correction of any single candidate will remove all the contradictions (see later).

Example 3.2: Consider Example 3.1 further. According to the "false" node of the ATMS and its label $\{\{R_1R_2, R_3R_4\}, \{R_1R_2, R_5R_6\}\}$, it is obvious that two minimal candidates can be generated: $C_1 = [R_1R_2]$ and $C_2 = [R_3R_4, R_5R_6]$, which means that fuzzy reasoning component R_1R_2 may be defective or that fuzzy reasoning components R_3R_4 and R_5R_6 may both be defective at the same time. This result can be better understood by examining the following:

- by $\perp_1$, at least one of $\{R_1R_2, R_3R_4\}$ is faulty;
- by $\perp_2$ or $\perp_3$, at least one of $\{R_1R_2, R_5R_6\}$ is faulty;
- by $\perp_4$ or $\perp_5$, at least one of $\{R_1R_2, R_5R_6, R_9R_{10}\}$ is faulty;
- by $\perp_6$, at least one of $\{R_1R_2, R_3R_4, R_7R_8\}$ is faulty;
- by $\perp_7$ or $\perp_8$, at least one of $\{R_1R_2, R_3R_4, R_5R_6, R_7R_8, R_9R_{10}\}$ is faulty.

What GDE deduces is that at least one of the following two sets of fuzzy reasoning components is faulty, $\{R_1R_2\}$ or $\{R_3R_4, R_5R_6\}$. The set $\{R_1R_2\}$ is considered as a candidate because R_1R_2 belongs to every contradiction given above and if it is faulty, all these five assertions are explained. Similarly, the set $\{R_3R_4, R_5R_6\}$ is considered as a candidate because if R_3R_4 and R_5R_6 are faulty simultaneously, they jointly explain all these assertions due to at least one element of $\{R_3R_4,\ R_5R_6\}$ belonging to each conflict listed above. Any other candidate is a superset of at least one of these two candidates and thus removed.

In terms of interpolation, that fuzzy reasoning component R_1R_2 is defective means that any interpolated rule whose antecedent is located between the antecedents of R_1 and R_2 is faulty and needs to be modified. That fuzzy reasoning components R_3R_4 and R_5R_6 are defective means that those interpolated rules whose antecedents locate between the antecedents of R_3 and R_4, and between those of R_5 and R_6 are faulty and need to be modified simultaneously. This leads to the development of the following procedure for modification of the generated faulty fuzzy reasoning components.

IV. Candidate Modification

Having described the method for minimal candidate generation, this section deals with how to correct such defective fuzzy reasoning components. It explores the presumption that any observed inconsistencies are dependent upon the found faults, which is assumed by GDE.

A. Consistency restoring algorithm

Because each single candidate explains the entire set of current contradictions, consistency can be restored by successfully correcting any single candidate. Intuitively, a candidate of the smallest cardinality has the highest likelihood to be the real culprit which may have caused all the detected inconsistencies. Therefore, candidate modification is always tried from a candidate containing the least elements (with any tie broken at random). Given a set of candidates, the proposed modification procedure is outlined as follows, and is illustrated in Figure 5.

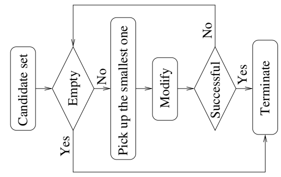

Fig. 5. Consistency restoring algorithm

Step 1. If the candidate set is not empty, randomly pick up a candidate of the smallest size and go to the next step; otherwise, terminate and return fail.

Step 2. Correct each defective fuzzy reasoning component in the candidate and propagate the modification to all the interpolated rules which are based on the defective component by the method given in the next section.

Step 3. If the modification is successful, that is all the contradictions are removed through the correction of each interpolated rule involved in this candidate, terminate and return success; otherwise, go back to Step 1.

The algorithm terminates under two situations. When the termination is caused by empty candidate set, it means that the modification fails and the proposed modification method is not suitable for the given problem. This may imply that the detected inconsistency may have been caused by incorrect observations or original rules given, which have been presumed to be true. Further modifications in this case remains as future research. However, when the termination is due to a successful modification, it means that consistency has been successfully restored and there is no need to try any other candidate.

Example 4.1: For the running example, there are two candidates in the candidate set. Because candidate C_1 is smaller than C_2 in cardinality, C_1 is chosen for modification first. Two rules have been interpolated based on this fuzzy reasoning component, both of which need to be modified: IR_1: If x_1 is A_{13}, then x_2 is A_{27}; IR_2: If x_1 is A_{14}, then x_2 is A_{28}.

B. Defective reasoning component correction

Inconsistencies result from the failure of interpolation (unless incorrect original observations or rules have been given, which are out of the scope of this paper). The reason for such failure roots in the presumption that the relationship between the antecedent variable and the consequent variable is linear. This linear assumption is reflected by the use of the same *relative placement factor* in the antecedent and consequent part of an interpolated rule (7). An intuitive way to remove the negative effect of this presumption is therefore, to shift the representative value of the consequence of a culprit rule within its domain to a "better" place, in an effort to explain all other propositions in the context. If so doing, the consequent value of the computed intermediate rule is changed with respect to the changing of the representative value of the consequence of the culprit interpolated rule. However, both move and scale rates that are generated by measuring the transformation from the antecedent of the intermediate rule to the antecedent of the interpolated rule remain intact. They are used to transform the consequence of the intermediate rule to the consequence of the modified interpolated rule. This ensures that the similarity between the antecedent of the intermediate rule and the antecedent of the interpolated rule is the same as that between the consequence of the intermediate rule and the consequence of the modified interpolated rule.

Based on these considerations, a set of simultaneous equations can be set up regarding all the interpolated rules which are dependent on the same defective fuzzy reasoning component, in order to modify their consequent values. The modification is carried out such that their corresponding propositions are $(1-\beta_0)$-consistent with the current context. The solution of these simultaneous equations forms the result of the modification. For convenience, in the rest of this paper, A^*_{ij} is used to denote the modified consequence of a culprit interpolated rule chose consequent value is A_{ij}, and ${A^*_{ij}}'$ and λ^*_{ij} are used to denote the corresponding modified intermediate rule consequence and the *relative placement factor* of A^*_{ij}, respectively. The following sub-sections address the requirements that the modification should satisfy.

1) Unique correction rate for rules interpolated from the same defective reasoning component: There may be more than one interpolated rule dependent on the same defective fuzzy reasoning component and all these interpolated rules should be accordingly modified along with the modification of the defective fuzzy reasoning component. That is, if an interpolated rule is altered because it depends on a defective fuzzy reasoning component, the similar alternation must also be applied to all other interpolated rules which depend on the same fuzzy reasoning component.

In this research, all those rules initially provided in the sparse rule base for interpolation are assumed to be fixed and true, and are referred to as base rules. Intuitively, the nearer any two rules are to each other, the more similar they are. Therefore, it is obvious that the interpolated rule whose antecedent is located farthest from both antecedents of a pair of base neighboring rules is the one that is most dissimilar to these neighboring rules. Thus, this farthest rule should be chosen for initial modification. In other words, the rule antecedent which sits in the middle most of the neighborhood of the two base rules is the one most likely to be wrong and needs to be modified the most. Any other interpolated rules dependent on the same fuzzy reasoning component can then be modified with reference to the modification of this one.

The modification of the interpolated rule which is located the middle most, is based on the presupposition that the relationship between the antecedent variable and the consequent variable is not strictly linear as assumed for interpolation. Suppose that the neighboring rules $A_{11} \Rightarrow A_{21}$ and $A_{1n} \Rightarrow A_{2n}$ are the two base rules used by a defective fuzzy reasoning component, that $A_{12}, A_{13}, ..., A_{1(n-1)}$ are observations or previously interpolated results located in between A_{11} and A_{1n}, and that A_{1j} $(2 \leq j \leq n-1)$ is the middle most one. It is interesting to observe that in carrying out interpolation, the presumed linear relation between an antecedent variable and the corresponding consequent variable is represented by a line in a coordinate plane (line P_0P_7 in Figure 6). The modification breaks this straight line segment P_0P_7 into two connected straight line segments P_0P_5 and P_5P_7 as illustrated in Figure 6. That is, it uses a first-order piecewise linear approximation to replace the original linear method. This gives one more degree of freedom in capturing the relation between the antecedent variable and the consequent variable than the use of just a single line.

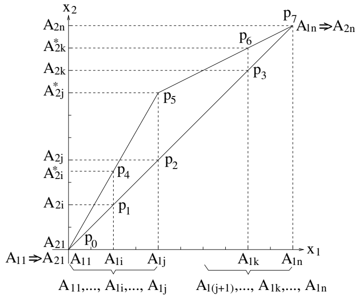

Fig. 6. Modification propagation through correction rate

The effect of this proposed modification method is to refine the defective fuzzy reasoning component by dividing it into two more accurate fuzzy reasoning components. In Figure 6, this corresponds to refining the fuzzy reasoning component represented by P_0P_7 into two fuzzy reasoning components represented by P_0P_5 and P_5P_7. From this, a pair of *correction rates* c^- and c^+ are introduced, denoted by (c^-, c^+). In particular, c^- represents the modification rate of those interpolated rules whose antecedents are on the left side of the antecedent value of the original (to be modified) interpolated rule (those from A_{12} to $A_{1(j-1)}$ in Figure 6), while c^+ represents the same meaning for those right located interpolated rules (those from $A_{1(j+1)}$ to $A_{1(n-1)}$ in Figure 6). The method for computing a correction rate pair is described below.

As illustrated in Figure 6, if the logical consequence of the middle most antecedent A_{1j} has been modified from A_{2j} to A^*_{2j} (i.e. from point p_2 to point p_5), the logical consequence of any antecedent A_{1i} located between A_{11} and A_{1j} is accordingly modified from A_{2i} to A^*_{2i} (i.e. from p_1 to p_4). That is, if the antecedent variable takes a value between A_{11} and A_{1j}, the interpolating mapping line (between the antecedent variable and the consequent variable) is modified from the line segment p_0p_2 to p_0p_5. For any given antecedent value A_{1i} lying between A_{11} and A_{1j}, the ratio of the distance between A_{21} and the modified consequence A^*_{2i} to the distance between A_{21} and the original unmodified consequence A_{2i} is a constant. It is this ratio that is represented by the correction rate c^-. c^+ is computed in exactly the same way, but replacing the left base rule consequence A_{21} to the right base rule consequence A_{2n}. Formally, the *correction rate* pair (c^-, c^+) is defined as:

$$\begin{cases} c^- = \frac{d(A_{21}, A^*_{2j})}{d(A_{21}, A_{2j})} = \frac{d(\mathrm{Rep}(A_{21}), \mathrm{Rep}(A^*_{2j}))}{d(\mathrm{Rep}(A_{21}), \mathrm{Rep}(A_{2j}))} \\ c^+ = \frac{d(A^*_{2j}, A_{2n})}{d(A_{2j}, A_{2n})} = \frac{d(\mathrm{Rep}(A^*_{2j}), \mathrm{Rep}(A_{2n}))}{d(\mathrm{Rep}(A_{2j}), \mathrm{Rep}(A_{2n}))}. \end{cases} \tag{20}$$

By (3) and (20), it follows that

$$\begin{cases} c^- = \frac{d(A_{21}, A^*_{2j})}{d(A_{21}, A_{2j})} = \frac{\frac{d(A_{21}, A^*_{2j})}{d(A_{21}, A_{2n})}}{\frac{d(A_{21}, A_{2j})}{d(A_{21}, A_{2n})}} = \frac{\lambda^*_{2j}}{\lambda_{2j}} \\ c^+ = \frac{d(A^*_{2j}, A_{2n})}{d(A_{2j}, A_{2n})} = \frac{1 - \frac{d(A_{21}, A^*_{2j})}{d(A_{21}, A_{2n})}}{1 - \frac{d(A_{21}, A_{2j})}{d(A_{21}, A_{2n})}} = \frac{1 - \lambda^*_{2j}}{1 - \lambda_{2j}}. \end{cases} \tag{21}$$

For any given antecedent A_{1i} $(2 \leq i \leq j-1)$, which is located on the left side of A_{1j}, its consequence A_{2i} is modified to A^*_{2i}, whose corresponding *relative placement factor* λ^*_{2i} satisfies:

$$\lambda^*_{2i} = \lambda_{2i} \cdot c^-. \tag{22}$$

Similarly, for any antecedent A_{1k} $(j+1 \leq k \leq n-1)$, which is on the right side of A_{1j}, the corresponding *relative placement factor* λ^*_{2k} of its modified consequence A^*_{2k} satisfies:

$$1 - \lambda^*_{2k} = (1 - \lambda_{2k}) \cdot c^+. \tag{23}$$

Example 4.2: In the running example, because fuzzy set A_{14} is located nearer the middle than A_{13}, the culprit interpolated rule IR_2 will be modified first. Suppose that the *relative placement factor* of the modified consequence is λ^*_{28}. Then the correction rate pair is:

$$c^- = \frac{\lambda^*_{28}}{\lambda_{28}}; \quad c^+ = \frac{1 - \lambda^*_{28}}{1 - \lambda_{28}}.$$

Accordingly, IR_1 should be modified with respect to the generated *correction rate* pair (c^-, c^+). The *relative placement factor* λ^*_{27} of the modified consequence satisfies:

$$\lambda^*_{27} = \lambda_{27} \cdot c^-.$$

The modified interpolated rule consequences A^*_{27} and A^*_{28} can thus be expressed as follows, where $j = 7, 8$:

$$A^*_{2j}{}' = (1 - \lambda^*_{2j}) A_{21} + \lambda^*_{2j} A_{22}; \quad T(A_{13}', A_{13}) = T(A^*_{27}{}', A^*_{27}); \quad T(A_{14}', A_{14}) = T(A^*_{28}{}', A^*_{28}).$$

2) Consistency of modified propositions: This requirement ensures that the consequence of each modified interpolated rule is at least $(1 - \beta_0)$-consistent with the current context. In accordance with the definition of contradictory degree (16), if the intersection point between two fuzzy sets is not lower than β_0, the contradictory degree between them is less than β_0. There is an equivalent way to represent β_0-contradiction by using β_0-*cut* due to the convexity of the fuzzy sets considered herein. If the intersection of β_0-cuts of two fuzzy sets is empty, the contradictory degree between them is higher than β_0.

This leads to the fact that the contradictory degree of fuzzy sets concerning a common variable can be calculated according to the given membership functions of these fuzzy sets. Suppose that m object values $A_{i1}, A_{i2}, ..., A_{im}$ are obtained for variable x_i. If they are $(1 - \beta_0)$-consistent, they must satisfy:

$$\bigcap_{j=1}^{m} (A_{ij})_{\beta_0} \neq \varnothing, \tag{24}$$

where $(A_{ij})_{\beta_0}$ denotes the β_0-*cut* of A_{ij}.

Example 4.3: For the running example, fuzzy sets A^*_{27} and A^*_{28} must satisfy the following constraints with respect to this requirement: $(A^*_{27})_{\beta_0} \cap (A^*_{28})_{\beta_0} \neq \varnothing$.

3) Consistency over modified proposition propagation: Every modified value of a given variable is propagated through all possible subsequent interpolations that depend on that variable, as dictated by the dependencies recorded by the ATMS. The corresponding propositions of such updated values are required to be $(1 - \beta_0)$-consistent. The propagation process follows the standard interpolation approach strictly. Similar to the last requirement, the contradictory situation is also checked through β_0-cut.

For simplicity, let function $I(A_{ij}, R_l R_r) = A_{kj}$ denote the standard interpolation from the antecedent fuzzy set A_{ij} to the consequent value A_{kj}, based on the fuzzy reasoning component involving the neighboring rules R_l and R_r. Suppose that m object values $A_{i1}, A_{i2}, ..., A_{im}$ of variable x_i are modified which are located between the antecedent values of rules R_l and R_r, that the corresponding modified object values of variable x_k are A^*_{kj}, $j \in \{1, 2, ..., m\}$, and that n object values A_{kl}, $l \in \{1, 2, ..., n\}$, of variable x_k are already obtained one way or another. If the modified consequences A^*_{kj} are all $(1 - \beta_0)$-consistent, then they must satisfy:

$$\begin{cases} A^*_{kl} = I(A^*_{ij}, R_l R_r) \\ \left(\bigcap_{j=1}^{m} (A^*_{kj})_{\beta_0} \right) \cap \left(\bigcap_{l=1}^{n} (A_{kl})_{\beta_0} \right) \neq \varnothing. \end{cases} \tag{25}$$

Example 4.4: Particularly, for the running example, the modified fuzzy sets A^*_{27} and A^*_{28} should be propagated forward to the corresponding object values of variables x_3, x_4 and x_5. The propagated object values of variable x_3 must satisfy the following equations simultaneously:

$$A^*_{35} = I(A^*_{27}, R_3 R_4); \; A^*_{36} = I(A^*_{28}, R_3 R_4); \; (A^*_{35})_{\beta_0} \cap (A^*_{36})_{\beta_0} \neq \varnothing.$$

Similarly, for the object values of variable x_4, they must satisfy:

$$A^*_{46} = I(A^*_{27}, R_5 R_6); \; A^*_{47} = I(A^*_{28}, R_5 R_6); \; (A^*_{46})_{\beta_0} \cap (A^*_{47})_{\beta_0} \cap (A_{45})_{\beta_0} \neq \varnothing.$$

Also, for the object values of variable x_5, the following equations need to be satisfied:

$$A^*_{55} = I(A^*_{35}, R_7 R_8); \quad A^*_{56} = I(A^*_{46}, R_9 R_{10});$$
$$A^*_{57} = I(A^*_{47}, R_9 R_{10}); \quad A^*_{58} = I(A^*_{36}, R_7 R_8);$$
$$(A^*_{55})_{\beta_0} \cap (A^*_{56})_{\beta_0} \cap (A^*_{57})_{\beta_0} \cap (A^*_{58})_{\beta_0} \cap (A_{55})_{\beta_0} \neq \varnothing.$$

4) Combination of correction requirement criteria: As described above, each requirement induces a set of constraining equations over the interpolation. For a detected inconsistency, all such induced equations must be satisfied simultaneously. If there exists at least one solution for these equations, the candidate is modified successfully. Otherwise, this candidate is discarded and the next one of the smallest cardinality will be tried as indicated in the algorithm given in Section IV-A.

Example 4.5: For the running example, with respect to candidate C_1, because there is no solution which satisfies all the equations listed above simultaneously, it is discarded. C_2 is then taken for tentative modification. Four rules have been interpolated through the two fuzzy reasoning components that comprises the candidate, which need to be modified:

IR_3: If x_2 is A_{27}, then x_3 is A_{35}; IR_4: If x_2 is A_{28}, then x_3 is A_{36};
IR_5: If x_2 is A_{27}, then x_4 is A_{46}; IR_6: If x_2 is A_{28}, then x_4 is A_{47}.

Because both fuzzy reasoning components $R_3 R_4$ and $R_5 R_6$ need to be modified and the result is irrelevant to the order of modifications, either $R_3 R_4$ or $R_5 R_6$ can be modified first. In this example, $R_3 R_4$ is arbitrarily taken to modify first. Following the requirement of Section IV-B.1, since A_{28} is located nearer the middle than A_{27}, the modification starts from the interpolated rule IR_4. Assume that the *relative placement factor* of the consequence of IR_4 is modified to λ_{36}^*, the *correction rate* pair (c^-, c^+) for the culprit fuzzy reasoning component R_3R_4 can be calculated as follows:

$$c_{R_3R_4}^- = \frac{\lambda_{36}^*}{\lambda_{36}}; \quad c_{R_3R_4}^+ = \frac{1-\lambda_{36}^*}{1-\lambda_{36}}.$$

The *relative placement factor* λ_{35}^* of the modified consequence of IR_3, A_{35}^* is computed according to (22), such that $\lambda_{35}^* = \lambda_{35} \cdot c_{R_3R_4}^-$. With such assumed *relative placement factors*, fuzzy sets A_{35}^* and A_{36}^* can be calculated as follows:

$$A_{35}^{*}{}' = (1-\lambda_{35}^*)A_{31} + \lambda_{35}^* A_{32}; \quad A_{36}^{*}{}' = (1-\lambda_{36}^*)A_{31} + \lambda_{36}^* A_{32};$$
$$T(A_{27}', A_{27}) = T(A_{35}^{*}{}', A_{35}^*); \quad T(A_{28}', A_{28}) = T(A_{36}^{*}{}', A_{36}^*).$$

For fuzzy reasoning component R_5R_6, because A_{27} is located nearer the middle than A_{28}, the modification starts from the interpolated rule IR_5. Similarly, assume that the *relative placement factor* of the consequence of IR_5 is modified to λ_{46}^*, then the following equations can be set for interpolated rules based on fuzzy reasoning component R_5R_6 according to the requirement of Section IV-B.1:

$$c_{R_5R_6}^- = \frac{\lambda_{46}^*}{\lambda_{46}}; \quad c_{R_5R_6}^+ = \frac{1-\lambda_{46}^*}{1-\lambda_{46}}; \quad (1-\lambda_{47}^*) = (1-\lambda_{47}) \cdot c_{R_5R_6}^+;$$
$$A_{46}^{*}{}' = (1-\lambda_{46}^*)A_{41} + \lambda_{46}^* A_{42}; \quad A_{47}^{*}{}' = (1-\lambda_{47}^*)A_{41} + \lambda_{47}^* A_{42};$$
$$T(A_{27}', A_{27}) = T(A_{46}^{*}{}', A_{46}^*); \quad T(A_{28}', A_{28}) = T(A_{47}^{*}{}', A_{47}^*).$$

Requirements given in Sections IV-B.2 and IV-B.3 ensure that the modified propositions and their propagation are $(1-\beta_0)$-consistent, which can be expressed as:

$$(A_{35}^*)_{\beta_0} \cap (A_{36}^*)_{\beta_0} \neq \varnothing; \quad (A_{46}^*)_{\beta_0} \cap (A_{47}^*)_{\beta_0} \cap (A_{45})_{\beta_0} \neq \varnothing;$$
$$A_{55}^* = I(A_{35}^*, R_7R_8); \quad A_{58}^* = I(A_{36}^*, R_7R_8);$$
$$A_{56}^* = I(A_{46}^*, R_9R_{10}); \quad A_{57}^* = I(A_{47}^*, R_9R_{10});$$
$$(A_{55}^*)_{\beta_0} \cap (A_{56}^*)_{\beta_0} \cap (A_{57}^*)_{\beta_0} \cap (A_{58}^*)_{\beta_0} \cap (A_{55})_{\beta_0} \neq \varnothing.$$

Solving these simultaneous equations leads to one solution which is illustrated in Figure 7. It is clear from this figure that there is no β_0-contradiction any more and thus consistency has been restored. This means that the original inconsistent interpolation process has been corrected with consistent interpolated results throughout.

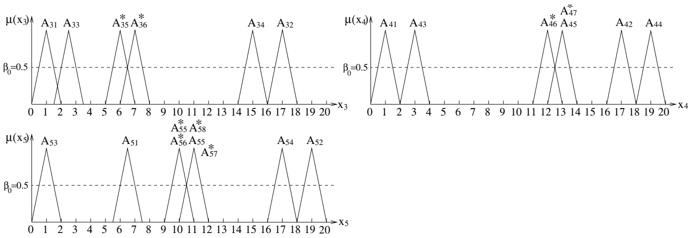

Fig. 7. The solution for the running example

V. Conclusions

This paper has made use of popular symbolic AI tools, ATMS and GDE to support fuzzy interpolation. ATMS records dependencies between interpolated rules and the neighboring rules employed for interpolation, while GDE generates minimal candidates with each of which explaining the whole set of contradictions in a given situation. The paper has further proposed a method to modify the identified culprit interpolated rules in an effort to restore reasoning consistency. This approach is built on the observation that the prerequisite to use fuzzy interpolation is the linear relationship between the antecedent variable and the consequent variable. The method works by first extracting the entire set of interpolated rules which depend on the same pair of neighboring rules in the generated candidate list. Then, it imposes a group of equations which not only constrain the modified propositions and ensure their propagation to be consistent, but also guarantee the original similarity-based reasoning in fuzzy interpolation to be followed. Finally, the approach corrects the culprit interpolated rules by solving the set of simultaneous equations. The working of this method is illustrated with a practically significant example. Although the work is built on the basis of scale and move transformation-based interpolation approach, the ideas should be readily transferrable to other approaches for fuzzy interpolation.

While the proposed approach is promising, further improvements may enhance its potential. First of all, only triangular fuzzy sets are considered in this paper. A natural extension is to deal with more complex fuzzy set representations, such as trapezoidal or bell-shaped. Also, it seems to be of great potential for the proposed method to be used in fuzzy extrapolation. In addition, it is worthwhile to investigate how to distinguish culprit candidates with the same size probably by exploring the differences in the consistency degrees. Further more, research on how to deal with rules with multiple antecedent variables is necessary since the present work only concerns single antecedent rules. Finally, all base rules which are provided in the initial rule base for interpolation are assumed to be totally true and are fixed. However, this is very difficult to be satisfied in many real-world problems despite it is a common assumption made in the literature of interpolative reasoning. Thus it is important to extend the proposed work to allow base rules to be modifiable as well.

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